Width of the Δ Resonance in Nuclei

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In this work we evaluate the imaginary part of the isobar Δ self-energy Σ_Δ from the two-body absorption process $\Delta + N \rightarrow 2N$. This contribution is calculated using a recently developed non-relativistic scheme, which allows for an evaluation of the self-energy with a basis of single-particle states appropriate for both bound hole states and for particle states in the continuum. In order to test the medium dependence of the self-energy, we calculate the two-body absorption term Σ_Δ^{A2} for several finite nuclei with $N = Z$, i.e. ^{16}O, ^{40}Ca and ^{100}Sn. The resulting self-energy, which is energy dependent and non-local, is compared with a simple parameterization derived from nuclear matter.

14.20.Gk, 21.60.-n,24.10.Cn

The self-energy of the Δ resonance is central in the understanding of nucleus-nucleus collisions, pion-nuclear processes and photonuclear reactions at intermediate energies, see e.g. Ref. [1] for a recent review. The self-energy Σ_Δ is given by a real part and an imaginary part

$$\Sigma_\Delta(\omega, k, k') = Re\Sigma_\Delta(\omega, k, k') + iIm\Sigma_\Delta(\omega, k, k'), \tag{1}$$

where ω and k, k' are the energy and momenta of the isobar, respectively.

In a microscopic derivation of the self-energy, it is common to represent various contributions to the self-energy by way of Feynman diagrams. Typical examples are shown in Fig. 1. Experimental data for pion scattering and absorption at various energies covering the isobar resonance region, suggest that the dominant process for absorption of pions at low energies is represented by diagram (a), which couples the isobar to two-nucleon one-hole states ($\Delta + N \rightarrow 2N$). Within the terminology of the isobar-hole model [2–4], diagram (b) is then supposed to represent the rescattering of e.g. a real pion, so-called reflection contributions to quasi-free scattering. Diagram (c) is an example of a self-energy contribution arising from three-body absorption mechanisms.

Unfortunately, there is no such thing as an experimental measurement of the Δ self-energy, although indirect information about the self-energy can be derived from e.g. pion-nucleus scattering. In the extensive analyses of Ref. [5], contributions arising from diagrams like (a) and (b) in Fig. 1, are represented by way of a Δ-spreading potential, fitted to provide best results for pion-nucleus elastic scattering.

In Ref. [6], a parameterization for the imaginary part of the Δ self-energy is obtained in nuclear matter by considering the contributions from the diagrams in Fig. 1, accounting thus for quasielastic corrections, two-body and three-body absorption. The nuclear matter results are then compared to the Δ-spreading potential from the empirical determination in Ref. [5] by allowing for a density dependent self-energy, given by the analytical expression

$$Im\Sigma_\Delta = Im\Sigma_\Delta^{A2} + Im\Sigma_\Delta^{A3} + Im\Sigma_\Delta^{Q}, \tag{2}$$

The term Q accounts for the quasielastic part while $A2$ is the two-body absorption part. $A3$ represents three-body absorption. A numerical parameterization for these terms was given by Oset and Salcedo [6], while analytical expressions based on the results of Ref. [6] were recently presented by Nieves *et al.* [7]. In the latter work the Σ_Δ^{A2} term of Eq. (2) has been calculated for a "typical" Δ, which is excited in the Δ-hole model, when a pion of a certain kinetic energy is absorbed. This parameterization reads

$$Im\Sigma_\Delta^{A2}(x) = Im\alpha_\Delta(x)\frac{1}{\beta(x)} arctg\left(\beta(x)(\rho/\rho_0)\right), \tag{3}$$

with $x = \frac{T_\pi}{m_\pi}$, T_π and m_π being the kinetic energy and mass of the pion, respectively. ρ is the density of particles and ρ_0 the corresponding density at nuclear matter saturation. Further, the function $Im\alpha_\Delta$ is

$$Im\alpha_\Delta(x) = -38.3\left(1 - 0.85x + 0.54x^2\right) \text{MeV}, \tag{4}$$

and

$$\beta(x) = 2.72 - 4.07x + 3.07x^2. \tag{5}$$

The parameterization of eq.(3) can be interpreted as the width of a Δ, which is excited when an energy $\omega = T_\pi + m_\pi$ is deposited at a nucleon in nuclear matter of density ρ.

The intention behind this brief report is to study the Σ_Δ^{A2} term directly for finite nuclei, since this contribution is expected to be the dominant one at energies below the isobar resonance [6]. For that purpose we will here employ a recently developed method to calculate the self-energy of the Δ in a finite nucleus. Microscopic calculations of e.g. the nucleon or Δ self-energy have preferentially been carried out in nuclear matter, the results of Ref. [6] being one example. One of the advantages of studies in nuclear matter is the possibility to describe the single-particle wave functions by plane waves. For a microscopic calculation in finite nuclei one has to take into account the fact that one needs different representations for bound hole states and particle states in the continuum. A method which allows for this has recently been developed [8,9]. The bound hole states are described in terms of harmonic oscillator (h.o.) wave functions while particle states are given by plane waves. The basic ingredients in our microscopic calculation of Σ_Δ^{A2} are briefly outlined below. The finite nuclei we consider are ^{16}O, ^{40}Ca and ^{100}Sn.

In order to calculate Σ_Δ^{A2}, we need first to define the transition potential $V_{NNN\Delta}$ for the $\Delta + N \rightarrow 2N$ process. For a nucleon and an isobar Δ interacting through the exchange of π plus ρ mesons, the transition potential $V_{NNN\Delta}$ is usually written, in the static non-relativistic limit, as [3]

$$V_{NNN\Delta}(\mathbf{k}) = -\left\{ D_\pi^{N\Delta}(\mathbf{k}) \frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} \boldsymbol{\sigma} \cdot \mathbf{k} \mathbf{S} \cdot \mathbf{k} + D_\rho^{N\Delta}(\mathbf{k}) \frac{f_{\rho NN} f_{\rho N\Delta}}{m_\rho^2} \boldsymbol{\sigma} \times \mathbf{k} \cdot \mathbf{S} \times \mathbf{k} \right\} \boldsymbol{\tau} \mathbf{T}, \tag{6}$$

with $S(T)$ the transition matrix which creates a spin (isospin) 3/2 object from a spin (isospin) 1/2 one. The meson propagator $D_{\pi,\rho}(\mathbf{k})$ is defined as in Ref. [9] as

$$D_{\pi,\rho}^{N\Delta}(\mathbf{k}) = \frac{1}{2} \left(\frac{1}{m_{\pi,\rho}^2 + \mathbf{k}^2} + \frac{1}{m_{\pi,\rho}^2 + \mathbf{k}^2 + m_{\pi,\rho}(m_\Delta - m_N)} \right). \tag{7}$$

The coupling constant for the π meson is given by a relation obtained from the non-relativistic quark model [10]

$$f_{\pi N\Delta} = \frac{6}{5}\sqrt{2} f_{\pi NN} = \frac{6}{5}\sqrt{2} g_{\pi NN} \frac{m_\pi}{2m_N}, \tag{8}$$

and similarly for the ρ meson we have

$$f_{\rho N\Delta} = \frac{f_{\pi N\Delta}}{f_{\pi NN}} g_{\rho NN} \frac{m_\rho}{4m_N} \left(1 + \frac{f_{\rho NN}}{g_{\rho NN}} \right), \tag{9}$$

with

$$f_{\rho NN} = \sqrt{4\pi} g_{\rho NN} \frac{m_\rho}{m_N} \left(1 + \frac{f_{\rho NN}}{g_{\rho NN}} \right). \tag{10}$$

In addition we include monopole form factors in order to regularize the potentials at short distances. The cutoff masses are $\Lambda_\pi = 1.2$ GeV and $\Lambda_\rho = 1.3$ GeV, while the coupling constants are $g_{\pi NN}^2/4\pi = 14.6$ and $g_{\rho NN}^2/4\pi = 0.95$, which are equal to the parameters which define the Bonn B nucleon-nucleon potential V_{NN} of table A.2 in Ref. [11]. Further, $\frac{f_{\rho NN}}{g_{\rho NN}} = 6.1$. The Bonn B potential is used to calculate the $G_{N\Delta}$-matrix, the next ingredient in our calculations. To calculate the $G_{N\Delta}$-matrix, we need first to evaluate the nucleon-nucleon G-matrix G_{NN}. This is done by solving the Bethe-Goldstone equation

$$G_{NN}(\Omega) = V_{NN} + V_{NN} Q \frac{1}{\Omega - QH_0Q} QG_{NN}(\Omega). \tag{11}$$

Here V_{NN} is the free nucleon-nucleon interaction. In this work V_{NN} is defined by the parameters of the Bonn B potential in table A.2 of Ref. [11]. The term H_0 is the unperturbed hamiltonian. This equation is solved with an angle-average nuclear matter Pauli operator Q with a fixed starting energy $\Omega = -10$ MeV and a Fermi momentum $k_F = 1.4$ fm^{-1}. From the nucleon-nucleon G_{NN} matrix, we can evaluate the $G_{N\Delta}$ matrix [9] through the relation

$$G_{N\Delta}(\Omega) = V_{NNN\Delta} + V_{NNN\Delta} Q \frac{1}{\Omega - QH_0Q} QG_{NN}(\Omega). \tag{12}$$

Having accounted for the short-range correlations through the introduction of the $G_{N\Delta}$-matrix, we are then able to set up the expression for the imaginary part of Σ_Δ^{A2}

$$\begin{aligned} Im\Sigma_\Delta^{A2}(j_b l_b k_b k_a \omega) = & -\frac{1}{2(2j_b+1)} \sum_{n_h l_h j_h} \sum_{JT} \sum_{lLS\mathcal{J}} \int k^2 dk \int K^2 dK \hat{J}\hat{T} \\ & \times \langle k_a l_b j_b n_h l_h j_h JT | \, G_{N\Delta} \, | klKL(\mathcal{J})SJT \rangle \\ & \times \langle klKL(\mathcal{J})SJT | \, G_{N\Delta} \, | k_b l_b j_b n_h l_h j_h JT \rangle \\ & \times \pi\delta(\omega + \varepsilon_h - \tfrac{K^2}{4M_N} - \tfrac{k^2}{M_N}), \end{aligned} \tag{13}$$

The single-hole energy ε_h is given by the eigenvalues of the harmonic oscillator minus a constant shift to place the Fermi energy at zero, while the energies of the particle states are represented by the pure kinetic energy. The variables k, K are the relative and center-of-mass momenta of the intermediate particle states p_1 and p_2 in Fig. 1. Further, l and L are the corresponding orbital momenta of the relative and center-of-mass motion. S, J and T are the total spin, total angular momentum and isospin, respectively. Finally, M_N is the average proton and neutron masses. A h.o. single-particle state is defined by the quantum numbers $n_h l_h j_h$, while plane waves are defined by $k_a l_a j_a$. For further details, see Ref. [9]. The energy variable ω refers to the energy of the Δ relative to the mass of a nucleon. Only positive energies ω contribute, as can be deduced from the δ function in Eq. (13).

To study the medium dependence of $Im\Sigma_\Delta^{A2}$, we evaluate Eq. (13) for the nuclei ^{16}O, ^{40}Ca and ^{100}Sn. The medium dependence of Eq. (13) is accounted for by the summation over single-hole states, represented by the $0s_{1/2}$, $0p_{1/2}$ and $0p_{3/2}$ single-hole states in ^{16}O, $0s_{1/2}$, $0p_{1/2}$, $0p_{3/2}$ $1s_{1/2}$, $0d_{3/2}$ and $0d_{5/2}$ single-hole states in ^{40}Ca and $0s_{1/2}$, $0p_{1/2}$, $0p_{3/2}$ $1s_{1/2}$, $0d_{3/2}$, $0d_{5/2}$ $1p_{1/2}$, $1p_{3/2}$, $0f_{5/2}$, $0f_{7/2}$ and $0g_{9/2}$ single-hole states in ^{100}Sn. Moreover, the oscillator parameters used in the calculation of the single-hole wave functions are 1.72 fm for ^{16}O, 2.04 fm for ^{40}Ca and 2.20 fm for ^{100}Sn.

In the discussion presented here, we only consider Δ isobar states with orbital angular momentum $l_b = 0$. A Fourier transformation of $Im\Sigma_\Delta^{A2}$ in Eq. (13) leads to an imaginary part, which depends on energy ω and is non-local in the coordinate r, the distance from the center of the nucleus. From the inspection of this function we observe that the non-locality is weak in the sense that it is different from zero only for distances r and r', which are close to each other. Therefore it makes sense to look at the local component of $Im\Sigma_\Delta^{A2}$ for the various energies as a function of the distance r [12]. As an example we present in the left part of Fig. 2 this local approximation obtained for ^{40}Ca. The shape of these functions is not really identical to a Woods-Saxon shape or a conventional density distribution. In particular at lower energies (ω below 200 MeV) one observes a clear surface contribution to $Im\Sigma_\Delta^{A2}$. Similar results are also obtained for the other nuclei (see right side of Fig. 2).

Finally, in Fig. 3, we compare the imaginary part of the Δ self-energy calculated in the local approximation for a typical radius of r=1.5 fm at various energies ω with the parameterization of Nieves et al. [7]. This comparison must be considered with some care. As discussed above, the parameterization of eq.(3) represents the average imaginary part of a Δ, which is typically excited, when a pion is absorbed in nuclear matter of density ρ, depositing an energy $\omega = T_\pi + m_\pi$. On the other hand, the results for finite nuclei show a non-trivial radial dependence and Fig. 3, just displays results for one "typical" radius. The nuclear matter parameterization is presented for a density ρ= 0.75 ρ_0 which is the average density of nucleons in ^{100}Sn. The agreement between the microscopic calculation for this nucleus and the parameterization, which is based on studies of nuclear matter is remarkable. This is true for both the absolute value as well as the shape of the energy-dependence. Only for the lightest nucleus which we considered, ^{16}O, the calculated width lies considerably below the parameterization.

In conclusion we would like to point out that our microscopic evaluation of the Δ-spreading potential in finite nuclei supports the parameterization of [7], which is based on studies of nuclear matter. For low energies, however, it may be important to consider a surface enhancement of the imaginary part in the self-energy of the Δ isobar.

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FIG. 1. Example of diagrams which arise in the evaluation of the self-energy of the isobar. (a) is the two-body absorption term evaluated in this work, (b) is an example of a so-called reflection contribution to quasi-free scattering while (c) stands for a three-body contribution. The wavy line represents the $G_{N\Delta}$-matrix, the single line is a nucleon while the double stands for the isobar Δ.

FIG. 2. Local representation of the imaginary part of $-\Sigma_\Delta^{A2}$ as function of the distance r from the center of the nucleus. In the left part of the figure results are shown for the nucleus ^{40}Ca, considering the energies ω = 100, 200 and 400 MeV. In the right part of the figure the results are displayed for the nuclei ^{16}O, ^{40}Ca and ^{100}Sn assuming an energy ω=300 MeV.

FIG. 3. Strength of the imaginary part of $-\Sigma_\Delta^{A2}$ at various energies. Results obtained for the local representation at r=1.5 fm, derived from the microscopic calculation of the Δ self-energy in the finite nuclei ^{16}O, ^{40}Ca and ^{100}Sn are compared to the parameterization of Nieves et al. [7], displayed in eq.(3) for a density $\rho = 0.75\rho_0$. This parameterization has been extrapolated from $\omega = m_\pi$ to ω=0.